**Hospital Readmission Reduction Strategies Using a Penalty-Incentive Model**

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**Abstract**

In 2012 the Centers for Medicare and Medicaid Services implemented a penalty-only system to reduce the number of hospitals with high readmission rates. We develop a penalty-incentive model for hospital readmissions in a basic game theoretic setting between an Insurer and Hospital. The model assumes that the probability of each patient’s readmission is linearly decreasing with the Hospital’s level of care, while the Hospital’s treatment cost for each patient is linearly increasing with the level of care. The Insurer aims to minimize their cost while the Hospital aims to maximize their revenue. The Insurer designs a penalty-incentive mechanism that can inspire the Hospital to adopt a proper level of care. The system is analyzed using centralized and decentralized control. We identify the Win-Win region for the penalty-incentive factor, in which both the Insurer and Hospital are better off under the proposed mechanism. Additionally, results show that the Win-Win region does not exist when a penalty-only system is used by the Insurer.

**Keywords**

hospital readmissions, health policy, game theory, win-win region

1. **Introduction**

A hospital readmission occurs when a patient is admitted to a hospital within 30 days after being discharged from an earlier hospital stay. The cost of hospital readmission has been high for many years [1]. To reduce high readmission rates, the Affordable Care Act (ACA) established the Hospital Readmission Reduction Program (HRRP). HRRP requires the Centers for Medicare and Medicaid Services (CMS) to reduce 30-day hospital readmissions. In 2013, CMS adopted a penalty-only mechanism for hospitals that performed below the risk-adjusted average. The readmission penalty was 1% in FY2013, 2% in FY2014, and was raised to 3% in FY2015. There are currently 5 conditions in which the readmission penalty is applied. In this paper, we focus on acute myocardial infarction (AMI).

According to [2], there are mainly three types of strategies to reduce hospital readmissions: service delivery reform, financing reform, and integrated service and financing reform. There are many resources, supporting or criticizing HRRP which discuss the effects and weaknesses of this program [4]. [5] develop a game-theoretic model and analyze the effectiveness of HRRP from an economic and operational perspective. As shown by [5], the HRRP does not always provide incentive for the Hospital to reduce the readmission rate, and competition between hospitals can even increase the non-incentivized hospitals.

In this paper, we develop a penalty-incentive mechanism that will reduce the hospital’s 30-day readmission rate by inspiring the hospital to adopt an improved level of care. Our solution approach is to derive the Insurer’s optimal penalty-incentive factor and the Hospital’s best response under an Insurer-lead Stackelberg setting assuming both agents are rational. We identify the Win-Win region for the penalty-incentive factor, in which both the Insurer and the Hospital are better off under the proposed mechanism.
Proposition 1. The centralized optimal level of care $y^*$ satisfies:

- If $\frac{d}{b} + 1 < \frac{1 + d}{e}$ then $y^* = 0$;
- If $\frac{d}{b} + 1 = \frac{1 + d}{e}$ then $y^* = 0$ or 1;
- If $\frac{d}{b} + 1 > \frac{1 + d}{e}$ then $y^* = 1$.

When $\frac{d}{b} + 1 > \frac{1 + d}{e}$, the optimal outcome of our mechanism reduces the system-wide cost, i.e., the system-wide benefit is improved. When $\frac{d}{b} + 1 \leq \frac{1 + d}{e}$, the set of desired actions can not efficiently improve the service efficiently, and thus,
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needs to be redesigned. In this paper we assume the set of actions is well-designed. Thus, we will only consider the case that $\frac{d}{e} + 1 > \frac{1+d}{e}$. Since we assume $e < d < 1$, $\frac{d}{e} + 1 > \frac{1+d}{e}$ actually implies $a > b$.

3.2 Decentralized Control

We assume rational agents and complete symmetric information. Under Insurer-lead decentralized control, M determines the value of $f$, and then $H$ determines the value of $y$.

Given the penalty/incentive factor $f$, $H$’s problem is to solve

$$\max \quad \Pi_H(y \mid f)$$

s.t. \quad $0 \leq y \leq 1$

where $\Pi_H(y \mid f)$ is given by (2). We denote the solution of the above problem by $y^*(f)$.

In order to determine the value of $f$, $M$ first predicts $y^*(f)$ and then solves the following problem

$$\min \quad TC_M(y^*(f), f) = \alpha[1-\phi_1 f + (\phi_1 + \phi_2)fy^*(f)](1 + d - ey^*(f))$$

s.t. \quad $0 \leq f \leq 1$

3.2.1 Hospital’s Level of Care

For simplification purposes, we use the following notations:

$$A = \frac{(\alpha - a)e + b(1 + d)}{\alpha[(\phi_1 + \phi_2)(1 + d) + \phi_1 e]}$$

(8)

$$B = \frac{(\alpha - a)e + b(1 + d - 2e)}{\alpha[(\phi_1 + \phi_2)(1 + d) - (\phi_1 + 2\phi_2)e]}$$

(9)

$$C = \frac{(\alpha - a)e + b(1 + d - e)}{\alpha[(\phi_1 + \phi_2)(1 + d) - \phi_2 e]}$$

(10)

Proposition 2. For any given $f$, $H$’s decentralized optimal solution for $y$ is given by

1. If $\alpha > a + \frac{\phi_1 b}{\phi_1 + \phi_2}$ then

$$y^*(f) = \begin{cases} 0, & 0 \leq f \leq A, \quad A < f < B \leq f \leq 1. \end{cases}$$

2. If $\alpha = a + \frac{\phi_1 b}{\phi_1 + \phi_2}$ then

$$y^*(f) = \begin{cases} 0, & 0 \leq f < \frac{\phi_1 b}{(\phi_1 + \phi_2)\alpha} \quad B \leq f \leq 1. \\ 1, & \frac{\phi_1 b}{(\phi_1 + \phi_2)\alpha} < f \leq 1. \end{cases}$$

3. If $\alpha < a + \frac{\phi_1 b}{\phi_1 + \phi_2}$ then

$$y^*(f) = \begin{cases} 0, & 0 \leq f \leq C, \quad C \leq f \leq 1. \end{cases}$$

3.2.2 Insurer’s Penalty-Incentive Factor

In this section we solve $M$’s problem in (7). Recalling Proposition 2 we need to consider three cases separately.

Case I: $\alpha > a + \frac{\phi_1 b}{\phi_1 + \phi_2}$ is the most complex case and has been omitted due to space limitations.

Case II: $\alpha = a + \frac{\phi_1 b}{\phi_1 + \phi_2}$ has the solution $f^* = \frac{b}{\phi_1 + \phi_2 \alpha}$. $y^*(f^*) = 1$, $TC_M(y^*(f^*), f^*) = (a + b)(1 + d - e)$ and $

\Pi_H(y^*(f^*), f^*) = 0$.

Case III: $\alpha < a + \frac{\phi_1 b}{\phi_1 + \phi_2}$ has the solution $f^* = C$, $y^*(f^*) = 1$, $TC_M(y^*(f^*), f^*) = \alpha(1 + \phi_2 C)(1 + d - e)$ and $

\Pi_H(y^*(f^*), f^*) = \Pi_H(1, C) = (\alpha(1 + \phi_2 C) - a - b)(1 + d - e)$. Note that in Case II and Case III, our mechanism can achieve centralized optimal solution since $y^*(f^*) = 1$. 
3.3 Do-Nothing Model

In the situation of do-nothing, we have \( f = y = 0 \). Denote H’s profit by \( \Pi^0_H \) and M’s cost by \( TC^0_M \) in this situation. Then we have

\[
\Pi^0_H = (\alpha - a)(1 + d) = (\alpha - a)em; \quad TC^0_M = \alpha(1 + d) = \alpha em.
\]

4. The Win-Win Region

The Win-Win region for \( f \) to achieve centralized optimization is given by

\[
\left[ \frac{1}{\phi_1} \left( \frac{(\alpha - a)e}{a(1 + d - e)} + \frac{b}{a} \right), \frac{1}{\phi_2} \left( \frac{e}{1 + d - e} \right) \right].
\]

Note that when \( \frac{\phi_1}{\phi_2} + 1 > \frac{1 + d}{e} \) we have \( \frac{(\alpha - a)e}{a(1 + d - e)} + \frac{b}{a} < \frac{e}{1 + d - e} \). Thus, the Win-Win region is not empty if \( \phi_2 > 0 \). In penalty-only case, we have \( \phi_2 = 0 \), the Win-Win region is empty. Given that the Win-Win region in not empty, M will always set \( f \) at the left boundary, i.e., \( f^* = \frac{1}{\phi_2} \left( \frac{(\alpha - a)e}{a(1 + d - e)} + \frac{b}{a} \right) \), and then H chooses \( y^*(f^*) = 1 \). Then, H will not lose, and M will get maximum benefit increase, while the centralized optimization achieves.

5. Numerical Analysis

The results of the numerical analysis of the penalty-incentive model are given in this section. The minimum, maximum, and average of for readimission probabilities, payment, and cost of 1738 hospitals for AMI were taken from a 3-year period as reported by [6] and are summarized in Table 1. This sample size includes all hospitals that had complete data entries. Since data omissions appear random, the authors assume this number is representative of the population.

<table>
<thead>
<tr>
<th>Probability ((d, e))</th>
<th>Payment ((\alpha))</th>
<th>Cost ((a, b))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>0.0606</td>
<td>$16656</td>
</tr>
<tr>
<td>Average</td>
<td>0.1871</td>
<td>$21865</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.4333</td>
<td>$32014</td>
</tr>
</tbody>
</table>

The first numerical analysis (NA-1) uses data for the current penalty-only model employed by CMS. The parameters chosen for the second numerical analysis (NA-2) are consistent with an incentive-only model. The numerical analysis results were computed using MATLAB R2015b.

5.1 NA-1

The first numerical analysis (NA-1) sets the model parameters to levels consistent with those found in the data and reflects the penalty-only policy where the incentive factor \( \phi_2 = 0 \) and the penalty factor \( \phi_1 = 3 \) represents the FY2015 penalty level. With 96.6% of the hospitals having readmission probabilities of 30% or less, this was taken as the upper-bound readmission probability of \( d \) while \( e \) was chosen such that the difference, \( d - e \), was equal to the minimum probability of readmission. The payment value \( \alpha \) was set to the average payment value from Table 1. The baseline cost \( a \) was set to the average cost and \( b \) was chosen such that the sum \( a + b \) was equivalent to the maximum cost from Table 1. A summary for all model parameters is given in Table 2.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Payment</th>
<th>Cost</th>
<th>Penalty-Incentive</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d = .3000 )</td>
<td>( \alpha = 21865 )</td>
<td>( a = 19512 )</td>
<td>( \phi_1 = 3 )</td>
</tr>
<tr>
<td>( e = .2494 )</td>
<td>( b = 11142 )</td>
<td>( \phi_2 = 0 )</td>
<td></td>
</tr>
<tr>
<td>( d - e = .0606 )</td>
<td>( a + b = 30654 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of a sensitivity analysis for parameters \( \phi_1 \) and \( \phi_2 \) are shown in Figures 1 and 2 respectively. The cost of the centralized and do-nothing models are given in the first row and the decisions of these respective models are given in the second row. Notice that the parameters in NA-1 meet the first condition in Proposition 1 and thus the parameters in NA-1 are not well-designed. As a result, the decentralized solution results are invalid. The figures indicate there is no Win-Win region for the current policy and model parameters because the centralized solution is always \( y = 0 \) for all values of \( \phi_1 \) and \( \phi_2 \).
The second numerical analysis (NA-2) considers an incentive-only policy. To satisfy conditions for a Win-Win region, the parameters were changed to those shown in Table 3 to create a set of well-designed actions and satisfy Case III for the Insurer. Specifically, $\alpha$ and $b$ were reduced. Additionally, $\phi_1$ was set to 0 for an incentive-only model and $\phi_2$ was set to 5 to represent a 5% incentive level. A summary for all model parameters in NA-2 is given in Table 3.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Payment</th>
<th>Cost</th>
<th>Penalty-Incentive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = .3000$</td>
<td>$\alpha=19511$</td>
<td>$a=19512$</td>
<td>$\phi_1=0$</td>
</tr>
<tr>
<td>$e = .2494$</td>
<td>$b=3902$</td>
<td>$\phi_2=5$</td>
<td></td>
</tr>
<tr>
<td>$d - e = .0606$</td>
<td>$a + b = 23414$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A Win-Win region in these figures if evident when the hospital H’s decision $y = 1$ in both the centralized and decentralized controls. In Figure 2, the Win-Win region only occurs when $\alpha < a$, which implies that the baseline cost to the Hospital is less than the Insurer payment. Thus, the hospital should incur even more costs and provide a higher level of care to receive a higher reimbursement. Finally, Figure 4 shows that the Win-Win region exists when the incentive level is high, or $\phi_2 > 5$.

6. Conclusions
We have developed a penalty-incentive model with conditions for a Win-Win region. The model and results show that the current policy is ineffective in inspiring an improved level of care. The numerical results also reveal a few preliminary observations. First, the payments $\alpha$ should be less than the baseline cost $a$. There is not a Win-Win region for a penalty-only model. There can only be a Win-Win region for a model that includes incentives $\phi_2 > 0$. Therefore, CMS should consider changing the policy from a penalty-only to an incentive-only model. In future research, we plan to analyze policy changes based on conditions other than AMI. We will also analyze policy changes based on hospital
characteristics (e.g., % of Medicaid patients, hospital size, etc.) to determine if certain hospitals do not benefit from the proposed changes to the readmission policy.

References